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OBSERVATIONS ON THE STRUCTURE OF A TURBULENT FREE CONVECTION BOUNDARY LAYER

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Abstract—The paper presents the results of experiments on the turbulent boundary layer formed adjacent to a heated vertical plate immersed in water. A brief presentation of gross features is followed by a detailed discussion of fluctuations and their attendant structural implications. Conclusions are drawn regarding the scale, intensity and intermittency of turbulence and the concept of a laminar sublayer.

NOMENCLATURE

- X, vertical distance from leading edge;
- Y, lateral distance from plate;
- y, normalized lateral distance, $10Y$;
- T, temperature;
- ΔT, overall temperature difference, $T_0 - T_\infty$;
- ϕ, normalized temperature difference, $\left(\dfrac{T - T_\infty}{T_0 - T_\infty}\right)$;
- U, velocity in X direction;
- u, normalized velocity, $U/2(\beta g X \Delta T)^{\frac{1}{2}}$;
- β, thermal expansion coefficient;
- g, gravitational acceleration;
- ν, kinematic viscosity.

Subscripts

- 0, plate;
- ∞, outside boundary layer.

INTRODUCTION

THE first systematic study of turbulent fall convection from a vertical surface appears to have been carried out by Griffiths and Davis [1], who made a detailed study of the velocity and temperature profiles near a flat isothermal surface suspended in air. Their work constituted the only known source of useful data for nearly 30 years. Recently, in a thorough study of the same problem, Cheesewright has presented results [2] which confirm and extend the earlier work.

Several other studies have been carried out with water [3–7]. In each of these, as in the above-mentioned air experiments, attention has been focused primarily on the mean velocity and temperature profiles. The gross boundary-layer features form a natural beginning for the experimentalist and are especially useful in substantiating the choice of profile in a semi-theoretical integral analysis [8–10].

Comparisons [2, 10] show that the experimental profiles obtained in both air and water do not accurately fit the simple profiles assumed in the various integral analyses. This, together with the apparent lack of any other form of information, suggests the need for a more detailed study of the boundary layer.

The work presented here is an attempt to reveal some of the basic features of a turbulent free convection boundary layer in water. It consists of a study of the local and overall structure of the mean boundary layer in terms of the fluctuation and frequency distributions and their relation to the mean profiles. The scale and intensity of turbulence will be discussed along with the concept of a laminar sublayer.

EXPERIMENTAL FACILITIES

The rig

The equipment [11] consisted of a stainless steel plate (14-in high, 6-in wide and 0·010-in thick) mounted in a large tank of water as shown in Fig. 1. The plate was kept straight

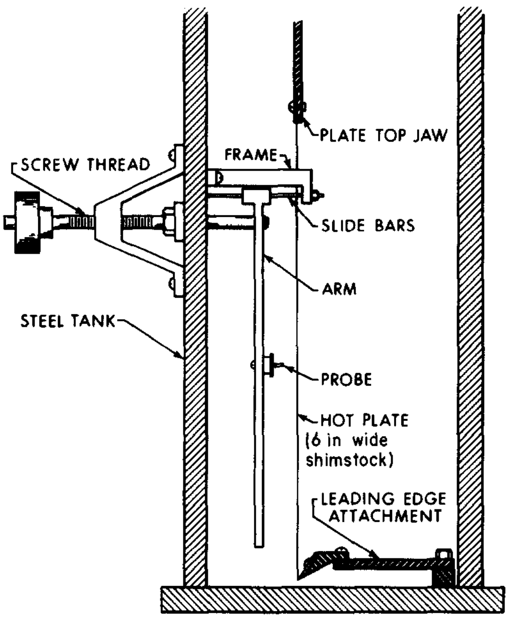

FIG. 1. Location of heated plate.

and vertical by means of a uniform tension applied between the accurately positioned upper and lower jaws. The upper jaw merely provided an anchorage but the lower jaw, which controls behaviour of the boundary layer near the leading edge, required careful design. The arrangement shown appeared to reconcile thermal, electrical and shape requirements and permitted a complete and well-defined boundary layer on one side of the plate. All measurements and visual observations were made on this side.

The plate was heated electrically by means of a high current transformer capable of providing 1000 A across 8 V. Typically, turbulent conditions required a current of about 800 A, corresponding to a power input to the plate of 7 kW. This power is, of course, completely dissipated in the surrounding water. To maintain a steady bulk water temperature within acceptable limits (e.g. changes less than 5 per cent of the temperature difference between the plate and the nominal bulk temperature) and yet restrict the volume of quiescent water required to within practical limits, a compromise had to be reached. To this end the dimensions of the tank were taken as 75 × 18 × 8 in, thus introducing a fairly large body of water above the plate and giving a period of about 5 min during which conditions were nominally steady. This was found to be barely sufficient for a complete run but was adequately extended by using a 3·5 kW capacity refrigerator cooling coil installed in the upper part of the tank and (for the velocity tests) switching off the current to the plate between each reading.

The tank was filled with water taken directly from the laboratory tap without any further preparation. For the purposes of the experiments the use of pure water was considered unnecessary since impurities do not likely affect the structure of a turbulent boundary layer. The possible error resulting from uncertainty of the precise value of the Prandtl number was ignored. De-aeration, on the other hand, was thought to be important because of the modifications within the boundary layer which growing and detached air bubbles may cause. To avoid these difficulties the water was vigorously heated before each test series. Visual observations through a window on one side of the tank showed that no bubbles were present during the experiments.

Instrumentation

The mean velocity profiles were determined by means of a quartz fibre anemometer similar to that used by Schmidt and Beckmann [12] for laminar flow. In distinction to the procedure of the above workers, no attempt was made to calibrate the anemometer experimentally. The principal reason for this is the difficulty encountered in producing a known turbulent flow in which the mean values, scale and intensity are similar to these anticipated in a free convection boundary layer. Casual observation showed the eddies to be large (comparable to the boundary-layer thickness) and slow moving (comparable to the mean velocities). Therefore it was decided to predict the velocity–deflection relation for the fibre by assuming that the eddies were much larger than the fibre (diameter) and moved uniformly past it. Using the drag-Reynolds number curve for a cylinder in a uniform stream [13] and the expression for the deflection of a simple, uniformly loaded cantilever, the tip deflection of the fibre was calculated in terms of the mean eddy velocity in the range of interest. Since the expression contained fluid properties a series of velocity–deflection curves had to be constructed, each for a different water temperature. Neglect of this temperature dependency would have introduced large errors, e.g. 50 per cent.

The fibre was mounted on the positioning arm shown in Fig. 1. Its nominal dimensions were 50 μ and 1·5 cm. Tip deflections, which were of the order of 0·01 cm, were measured by means of a cathetometer using a null method. The deflection was determined from the mean of several readings in each of which the image of the deflected tip was returned to the centre of reference cross wires by a fine vernier adjustment of the cathetometer carriage. The deflection error incurred in this method is estimated at about 10 per cent for the laminar readings but for turbulent conditions, in which the fibre fluctuations were of the same order of magnitude as the mean fibre deflection, errors of more than 20 per cent had to be tolerated in regions of very low velocity, e.g. near the plate. Although these errors appear rather high it may be noted that they do not materially affect the shape of the velocity profile and hence the conclusions drawn from this shape.

Temperatures were measured throughout by 0·010-in dia. iron–constantan thermocouple wires. Several of these were placed strategically within the tank and were used principally to monitor the bulk water temperature and its uniformity. One pair of wires was butt-welded to form a probe thermocouple which was mounted on the same arm (shown in Fig. 1) as the anemometer fibre, though not simultaneously. The rise time of the probe was measured in an auxiliary experiment and found to be less than 0·1 s. The probe signal was fed into one of several recorders; a Leeds and Northrup Speedomax G strip recorder (with each channel connected to the probe), a Varian strip recorder and a Leeds and Northrup potentiometer. The above combination proved most effective in producing the required data.

The plate temperature was determined from the strip chart records taken with the probe in contact with the plate. The maximum temperature thus recorded (which appeared as a series of short plateaux) was taken as the value at the plate. Short periods during which the electrical power supply was shut off resulted in no substantial change in the values obtained.

It was mentioned earlier that the plate was heated electrically by virtue of a high current transformer. The power delivered to the plate was measured at the busbars. The current flowing, as detected by a current transformer, was fed into a wattmeter which was also connected directly across between the busbars.

RESULTS AND DISCUSSION

Gross features

The development of the thermal boundary layer from laminar, through transition, to turbulent conditions may be followed by referring to the mean temperature profiles shown in Fig. 2. These clearly show the steepening of the profile as the flow becomes progressively more turbulent. The unnormalized temperature scale results in an apparently unvarying boundary-layer thickness of about 0·20 in. However, this is misleading because a normalized scale reveals that the thickness, arbitrarily taken as the "99 per cent point", varied noticeably. The thickness for the laminar results shown was

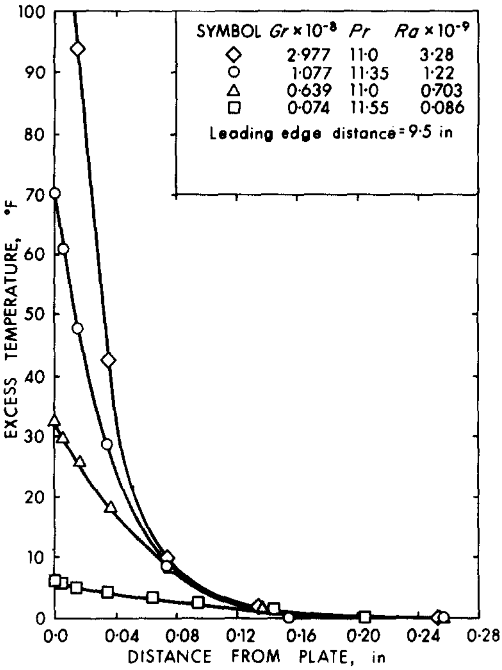

FIG. 2. Laminar, transition and turbulent temperature profiles.

35 per cent greater than that for the most turbulent conditions. Visual observations showed that the intermediate curves were not completely turbulent as the corresponding Rayleigh numbers (based on the bulk temperature) suggested.

Turbulent temperature profiles were obtained for the following range of conditions: 9·5 in $\leqslant x \leqslant$ 12·1 in, 9·8 $\leqslant Pr \leqslant$ 11·85, 1·44 $\leqslant Gr \times 10^{-8} \leqslant$ 8·4 and 1·71 $\leqslant Ra \times 10^{-9} \leqslant$ 8·36. These results are combined in Fig. 3 in which the normalized temperature is shown plotted against distance from the plate, also normalized for convenience. It is interesting to note that within the limitations imposed by scatter there does not appear to be any trend separating each set of points. Plots using other coordinates, the laminar similarity variable [14]

$$\eta = \left(\frac{\beta g\,\Delta T X^3}{4\nu^2}\right)^{\frac{1}{4}} \frac{Y}{X}$$

in particular, did not improve the correlation and often made it worse.

The development of the momentum boundary layer is presented in Fig. 4 which shows transition and turbulent velocity profiles at a fixed distance from the leading edge and one laminar profile taken at about half of this distance. It is evident from the figure that the velocity measurements are less quantitative than the temperature

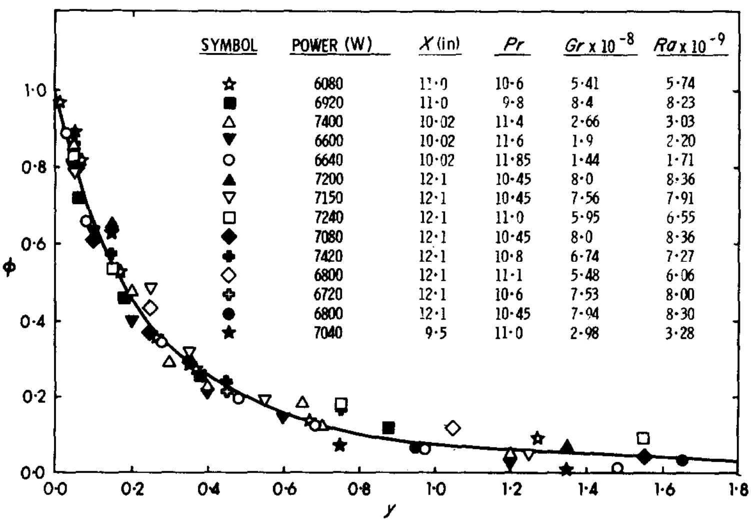

FIG. 3. Mean temperature profile.

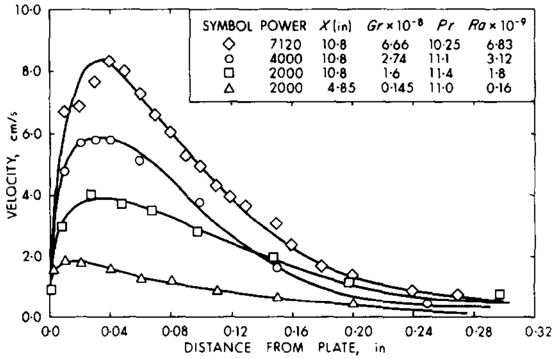

FIG. 4. Laminar, transition and turbulent velocity profiles.

measurements: between the plate and the velocity maximum large errors in the few available points reduces them to a qualitative role.

Figure 5 combines the results of a number of velocity tests run under fully turbulent conditions. The independent variable is the same as that used in Fig. 3 for the temperature profile and the velocities have been normalized by using the laminar expression, i.e. $u = U/2\,(\beta g X\,\Delta T)^{1/2}$.

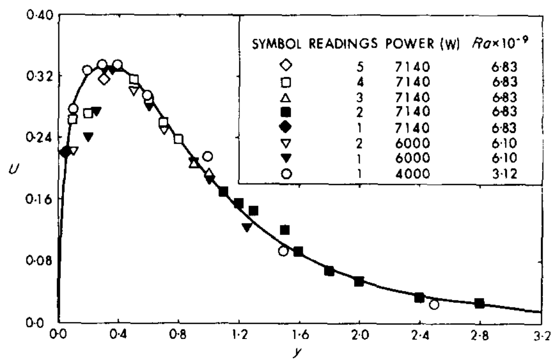

FIG. 5. Mean velocity profile.

Bearing in mind the uncertainty in the readings the correlation is evidently adequate for the range of conditions indicated in the figure.

It is interesting to compare the momentum layer thickness with the corresponding thermal layer thickness shown in Fig. 3. Their ratio is approximately equal to the square root of the Prandtl number; that is, the same relation that applies in laminar flow.

Structural details

A typical mean temperature profile selected from the above results is shown in Fig. 6 along with the corresponding velocity profile. Also plotted are the upper and lower limits of the temperature fluctuations. It is immediately

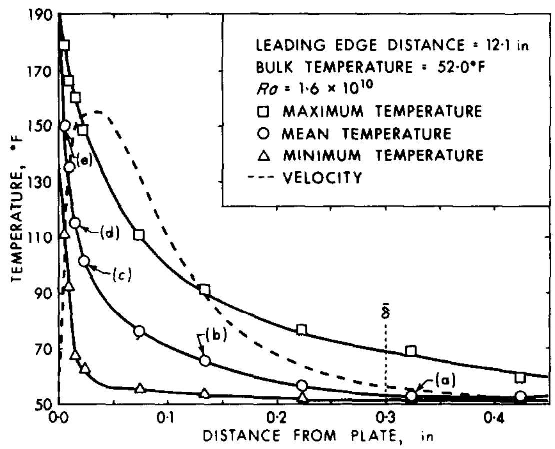

FIG. 6. Mean and fluctuating temperatures.

apparent that the fluctuations are of the same order as the mean values and therefore the widely used approximation that fluctuations are much smaller than mean values is not valid in free convection, at least for this range of Rayleigh numbers. Over most of the central portion of the boundary layer the fluctuation excess is more or less equal to the deficit and this suggests a certain symmetry in the behaviour of the eddies. In the outer region, which extends up to and beyond the position of the mean boundary-layer thickness ($\bar{\delta}$), excesses are significantly greater. This is to be expected since hot eddies from this region of the boundary layer are approaching and penetrating a nominally quiescent cool region. Immediately adjacent to the plate the reverse situation occurs. In this region, which occupies an extremely small fraction of the boundary layer, relatively cool eddies sporadically penetrate to the plate where the temperature is fixed at the highest value in the system.

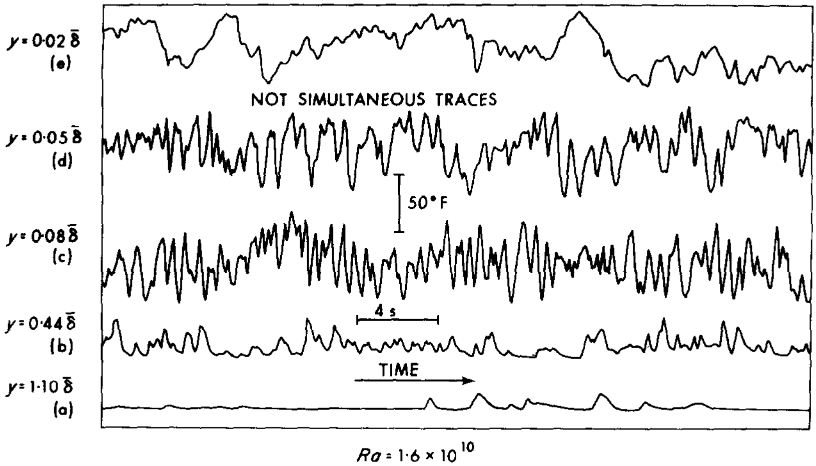

FIG. 7. Temperature fluctuation forms.

Continuous strip chart records taken at five locations throughout the boundary layer illustrate the variations in form of the fluctuations: these are shown in Fig. 7. The traces taken at locations (a) and (b) reveal the asymmetry of the turbulent process in the outer half of the boundary layer and beyond. In the region of the velocity maximum the fluctuations are typified by traces (c) and (d) which reveal an increase in intensity and greater symmetry. Trace (e) is near the inner boundary of this latter region and shows the damping effect of the wall but only a small change in intensity. This trace was taken with the thermocouple touching the plate, i.e. with a mean position less than 2 per cent of the boundary-layer thickness. Notwithstanding the error introduced by the thickness of the thermocouple itself, it is difficult to imagine a region immediately adjacent to the plate which is steadily laminar.

The traces shown in Fig. 7 were processed* to give frequency distributions about the mean values: the results are shown in Figs. 8 and 9. Near the velocity maximum (Fig. 8) the distributions exhibit the degree of symmetry that one might expect in a region of low velocity gradient, although the slight right-hand skew in the curve derived from trace (d) does reflect a slight wall effect. The left-hand skew anticipated from traces

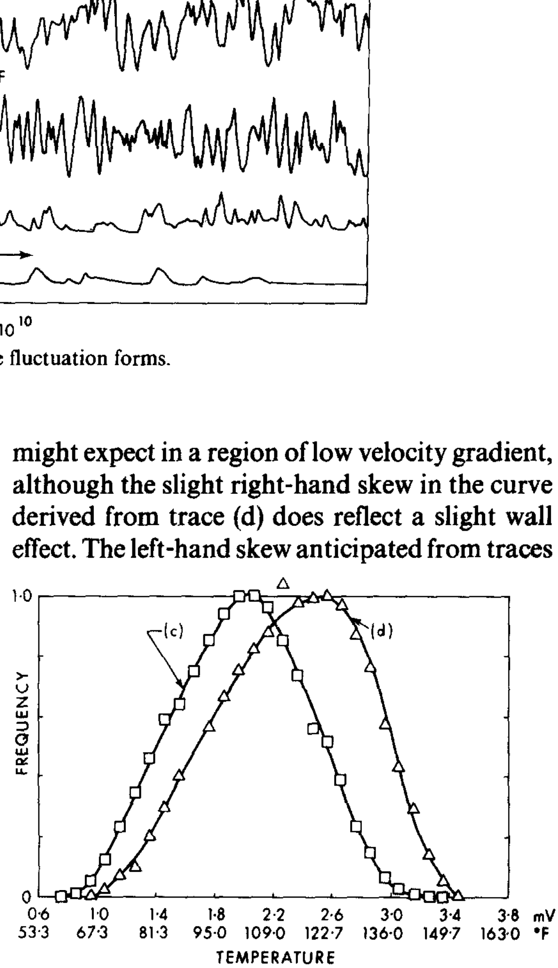

FIG. 8. Frequency distributions (near velocity maximum).

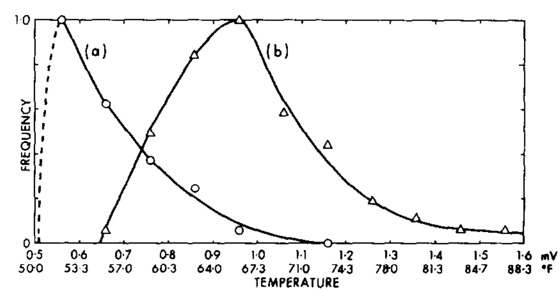

FIG. 9. Frequency distributions (outer region).

* The counts were normalized with respect to the mean value count.

(a) and (b) is clearly evident in Fig. 9. Curve (a) in particular exhibits almost discontinuous behaviour at the mean value, though it should be added that the trace becomes very difficult to interpret at this point.

The nature of the anemometric technique employed made it virtually impossible to study velocity fluctuations in the same manner as the temperature fluctuations. Visual observation of the fibre indicated that fluctuations were of the same order as the mean values and appeared to confirm the broad conclusions drawn from temperature records. In particular, exploration of the region immediately adjacent to the heated plate gave no evidence of a laminar region. Oscillations of form similar to those observed in the central region of the boundary layer were evident when the fibre was nominally in contact with the plate. Since the natural frequency of the fibre is well above the frequencies observed it was clear that the movements were driven by eddy motion. These observations were made in a region extending about 50 μ from the plate and therefore confirm and extend the observations made with the probe thermocouple.

A survey of intermittency throughout the boundary layer was also conducted and the results are shown in Fig. 10 in which the intermittency factor, defined as the fraction of time over which the flow was identifiable as turbulent, is plotted for a range of Rayleigh numbers. This reveals that the inner 40 per cent of the mean

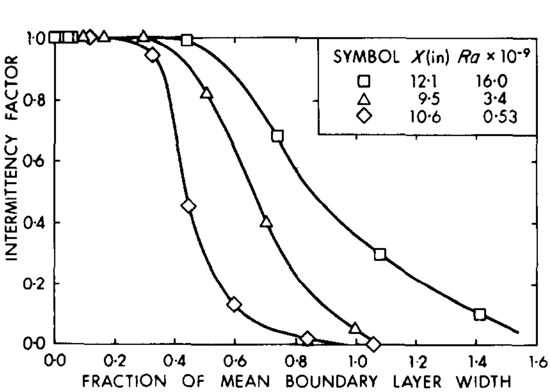

FIG. 10. Intermittency distribution.

boundary layer is turbulent from the point of transition upwards in Rayleigh number. It also shows that the extent of turbulence in the remaining portion of the boundary layer appears to be a monotonically increasing function of the Rayleigh number. A Rayleigh number of 3×10^9 was found to correspond to partial turbulence in the entire boundary layer, but only just. This is emphasized by the fact that the intermittency factor is small but non-zero at the edge of the boundary layer. The highest Rayleigh number indicated is about an order of magnitude above the transitional value but this still does not correspond to a completely turbulent layer as the figure shows. It is reasonable to expect the trend indicated to continue at higher Rayleigh numbers. This in turn suggests that eddies are capable of penetrating to distances of two boundary-layer widths from the plate and therefore that the scale of turbulence in the outer regions of a fully turbulent layer is comparable with the boundary-layer width. That is, the eddy diffusivity distribution in the outer region asymptotically approaches that accepted in the corresponding free jet problem [15].

CONCLUSIONS

The above observations indicate that for Rayleigh numbers (based on bulk conditions) less than $1{\cdot}6 \times 10^{10}$ the structure of a turbulent free convection boundary layer is quite different from the corresponding forced layer. The most striking difference is in the scale and intensity of turbulence. Thermocouple and anemometer measurements along with direct visual observation revealed that the scale of turbulence was of the order of the boundary-layer thickness in the outer region and decreased sharply near the heated plate. The intensity of turbulence, defined in terms of maximum mean values (since no obvious reference quantity is generally available), is of the order of 50 per cent throughout most of the boundary layer. At the edges of the mean boundary layer the intensity was noticeably less than this but still non-zero.

This last observation when considered at very small distances from the heated plate throws considerable doubt on the validity of the widely used concept of a laminar sublayer. It must be conceded that no measurements were made at the plate surface and therefore no direct conclusions can be drawn for that location. However, the existence of a laminar sublayer in water would require it to develop with unprecedented steepness in a region very much thinner than has been assumed in various integral analyses.

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REFERENCES

1. E. GRIFFITHS and A. H. DAVIS, The transmission of heat by radiation and convection, Food Investigation Board, Special Report No. 9, Dept. Sci. Ind. Res., London (1931).
2. R. CHEESEWRIGHT, Turbulent natural convection from a vertical plane surface, Ninth A.S.M.E.–A.I.Ch.E. Heat Transfer Conference, Seattle, A.S.M.E. Paper No. 67-HT-17 (1967).
3. P. J. WAIBLER, Natural convection heat transfer with water at high Grashof number, Ph.D. thesis, University of Illinois (1958).
4. M. S. VAN DYKE, Boundary layer temperatures in natural convection flow of water, M.S. thesis, University of Washington (1960).
5. T. FUJII, Experimental studies of free convection heat transfer, *Bull. J. S.M.E.* **2**(8), 551 (1959).
6. J. R. LARSON, Turbulent free convection in near critical water, Ph.D. thesis, Purdue University (1964).
7. J. R. LARSON and R. J. SCHOENHALS, Turbulent free convection in near-critical water, A.S.M.E.–A.I.Ch.E. Heat Transfer Conference, Los Angeles, A.S.M.E. Paper No. 65-HT-57 (1965).
8. E. R. G. ECKERT and T. W. JACKSON, Analysis of turbulent free convection boundary layer on flat plate, N.A.C.A. Report 1015 (1951).
9. F. J. BAYLEY, An analysis of turbulent free-convection heat transfer, *Proc. Instn Mech. Engrs* **169**(20), 361 (1955).
10. T. FUJII, An analysis of turbulent free convection heat transfer from a vertical surface. *Bull. J.S.M.E.* **2**(8), 559 (1959).
11. F. J. DEB. TROTTER, Study of turbulent free convection near a vertical plate, M.Sc. Thesis, University of Alberta (1967).
12. E. SCHMIDT and W. BECKMANN, Das Temperature und Geschwindigkeitfeld von einer. Warme abgebenden senkrechten Platte bei naturlicher Konvektion, *Tech. Mech. Thermo-Dynam Berl.* **1**(10), 341–349; **1**(2), 391–406 (1930).
13. H. SCHLICHTING, *Boundary Layer Theory*, p. 16. McGraw-Hill, New York (1960).
14. S. OSTRACH, An analysis of laminar natural-convection flow and heat transfer about a flat plate parallel to the direction of the generating body force, N.A.C.A. Rep. 1111 (1952).
15. W. SCHMIDT, Turbulente Ausbreitung eines Stromes erhitzer Luft, *Z. Angew. Math. Mech.* **21**, 265 (1941).

Résumé—L'article présente les résultats d'expériences sur la couche limite turbulente formée près d'une plaque plane verticale immergée dans l'eau. Une brève présentation des caractéristiques générales est suivie d'une discussion détaillée des fluctuations et de ce qu'elles entraînent au point de veu de la structure. On en tire des conclusions concernant l'échelle, l'intensité et l'intermittence de la turbulence et le concept d'une sous couche laminaire.

Zusammenfassung—Die Arbeit liefert Ergebnisse von Versuchen über die turbulente Grenzschicht an einer beheizten, senkrechten in Wasser getauchten Platte. Einer kurzen Wiedergabe der wesentlichen Merkmale folgt eine detaillierte Diskussion von Schwankungen und der damit zusammenhängenden strukturellen Folgerungen. Schlüsse werden gezogen hinsichtlich der Grösse, Intensität und Wechsel, die Turbulenz und dem Begriff der laminaren Unterschicht.

Аннотация—В статье представлены результаты экспериментов по турбулентному пограничному слою, образующемуся на нагретой вертикальной пластине, погруженной в воду. Дается краткое описание общей картины, а затем приводится подробное обсуждение её структурных особенностей. Сделаны выводы о масштабе, интенсивности и перемежаемости турбулентности, а также вводится понятие ламинарного подслоя.